

## Kwok's Cubes

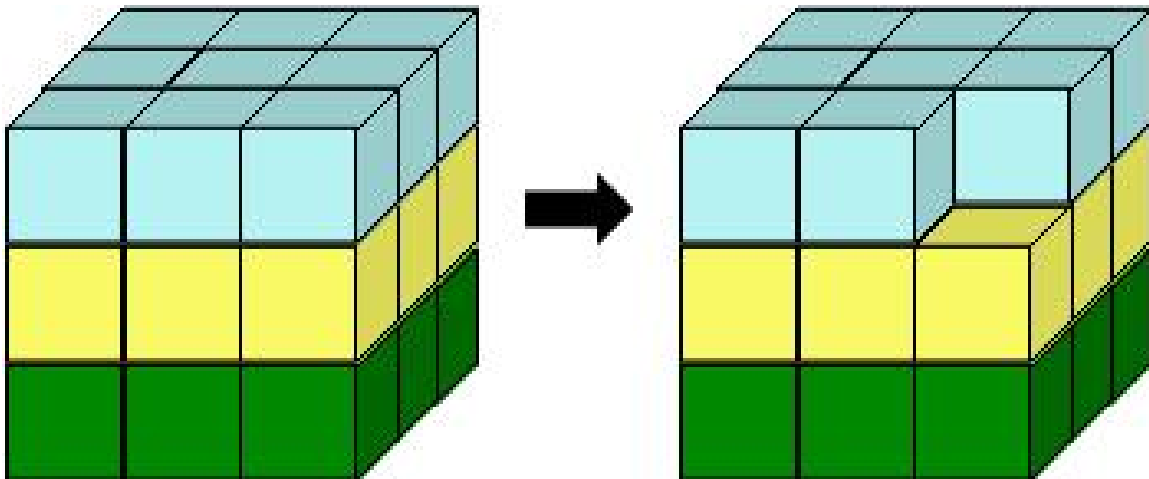
---

During an art lesson, a student created a cube of length 3 units by gluing a number of unit cubes together. While painting the cube, some unit cubes dropped out accidentally. The student, however, decided to continue painting the remaining figure.

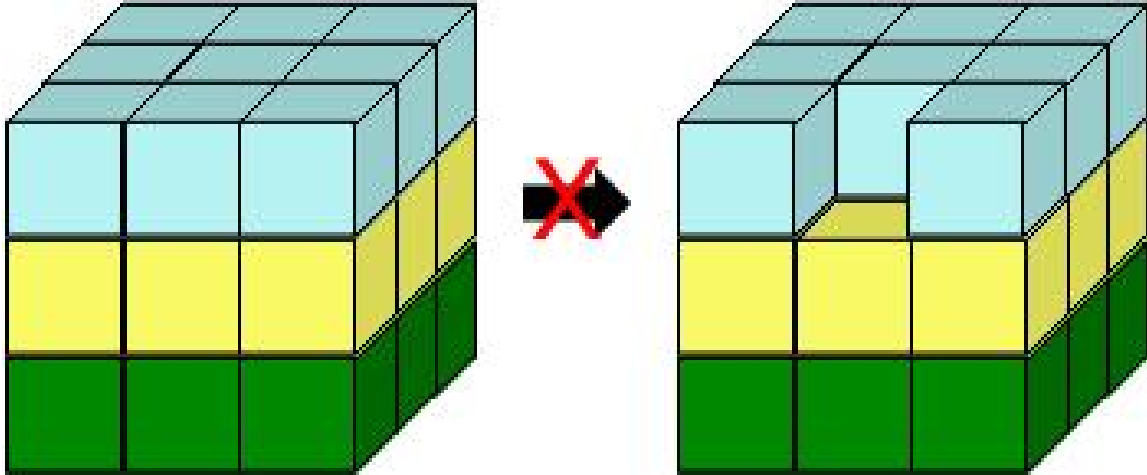
Assuming he used an equal amount to paint each face of the solid, he found that he still had to use the same amount of paint for painting the cube to paint the remaining solid figure. In other words, the solid figure had the same number of faces as the cube even though it had a smaller volume.

To satisfy his curiosity, the student continued to remove the unit cubes one at a time and counted the number of faces left in the remaining solid figure. To his surprise, he found that the solid figure still retained the same number of faces as the original cube after each removal of a unit cube.

For instance, removing the top corner cube leaves the same number of faces.



Removing the cube removed below adds an extra face to be painted.



What is the maximum number of unit cubes that can be removed from a cube of side 3 units long so that the surface area of the remaining solid figure is the same as the cube?

Draw a picture of the remaining solid figure.

**Challenge:**

What is the maximum number of unit cubes that can be removed from a cube of side 4 units long so that the surface area of the remaining solid figure is the same as the cube?

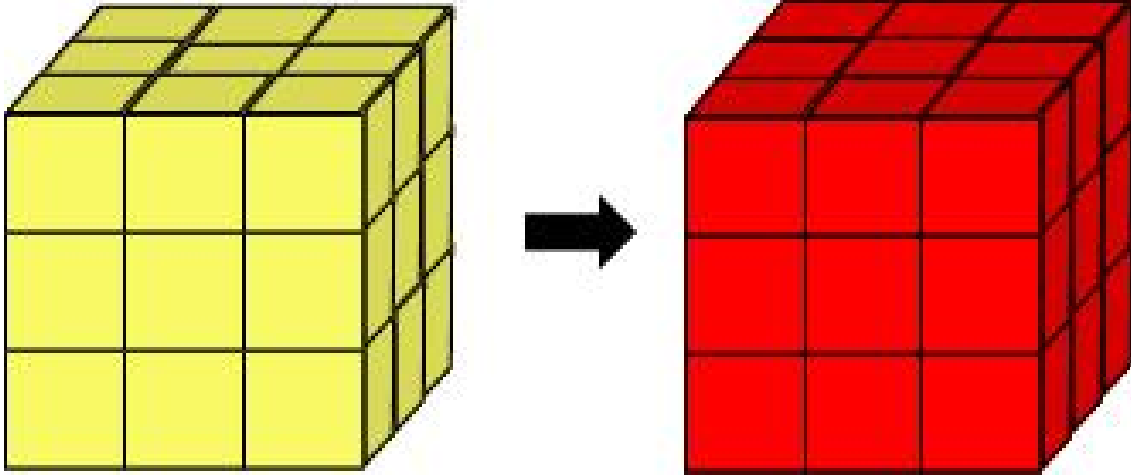
What is the maximum number of unit cubes that can be removed from a cube of side 5 units long so that the surface area of the remaining solid figure is the same as the cube?

What is the maximum number of unit cubes that can be removed from a cube of side  $n$  units long so that the surface area of the remaining solid figure is the same as the cube?

Credits: Henry Kwok, Singapore

**Extension:**

This extension is the classic painted cube problem. In the same class another student glued a cube of length 3 units by gluing a number of yellow unit cubes together. Then the student painted the outside of the cube red.



How many of the cubes have 1 red face? 2 red faces? 3? 4? 5? No red faces? The unit cube below has 2 red faces.

