

Grasshopper Jump-fest

One of the sports that grasshoppers play when they are by themselves is the jump-fest. To play, a number of grasshoppers stand on a tire tread:



Every few seconds all of the grasshoppers on one bit of the tire tread jump a number of tread-spaces equal to their number... so one grasshopper would jump one space... two grasshoppers would jump two spaces and so on... Any jump is permitted so long as it ends on top of another grasshopper and not an empty bit of tire tread.



first jump



second jump



third jump



final jump

The grasshoppers all win if they can all stack onto the same space.

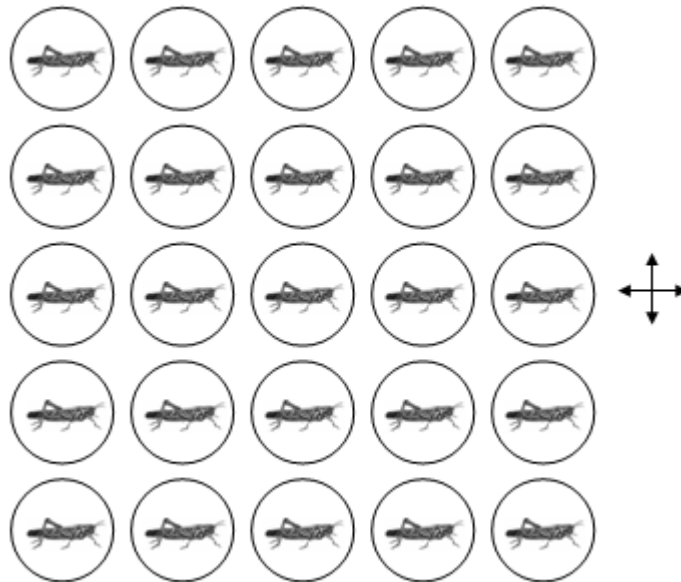
Can the grasshoppers be successful when 6, 7 or 8 of them play?

How many jumps does it take to successfully complete a game?

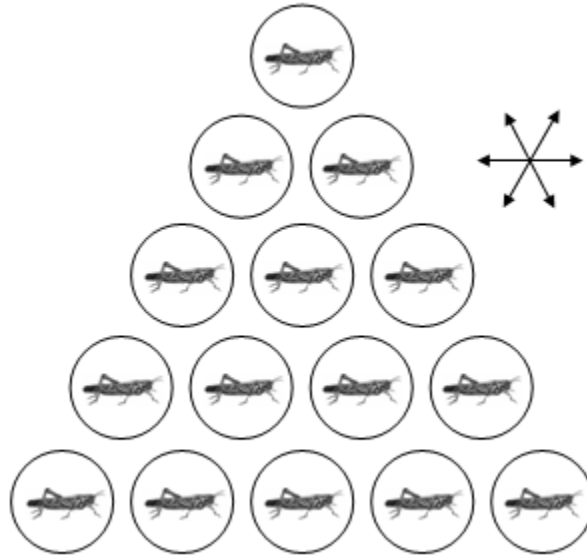
Find a pattern of jumping that works for any number of grasshoppers, or find a number of grasshoppers for which winning is impossible.

Extensions:

- When the King grasshopper plays, he just sits and expects the other grasshoppers to jump on top of him... In an 8 person game is it possible for the King to sit anywhere and still be successful?
- Is success always possible when playing with the King?
- One day a lizard suggested that 25 grasshoppers should play on a 5 by 5 square of twist-off bottle caps where the jumping can be vertical or horizontal. The lizard suggested that if they are successful, he wouldn't eat them any more. Will the lizard have to give up his diet rich in grasshopper nutrients?



- Enjoying the festive activities, and overhearing the lizard, a slithering snake suggested that if the grasshoppers could successfully stack themselves starting from the triangle shown, then he would gladly eat the lizard for them. What became of the lizard?



- What if one of the grasshoppers in this triangular arrangement was the King?
- Jix, an anti-social grasshopper never plays basic (tire-tread) jump-off, but laughs every time his fellow grasshoppers screw-up and fail to make a single stack. What is the fewest number of moves that Jix must wait for his fellow grasshoppers to be unable to move when 5, 6, 7 or 8 of them play?
- Jix suggests another sport: How many grasshoppers in total can stack onto X tire tread bits so that no jumps are possible, but that if you release the condition that a stack cannot jump to an empty space, then all stacks can jump. (Jix can fit 40 grasshoppers on 10 tread.)

Credits:

Theme, Problem and Extensions by Gord! © 2005 Galileo

The inspiration for grasshopper jumpfest came from the game Dvonn, the winner of multiple awards internationally. The game's designer, Kris Burm, is called Jix by his children.